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ON THE COINAGE RATIO OF SILVER AND GOLD.

By Alex. S. Christie, LL. M.,

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I purpose in this paper to determine, by a rigorous computation, based upon the essential elements of the question, the just variations in the commercial ratio of silver to gold, and to indicate how, and to what degree, these variations should affect the coinage ratio. The problem is, in many of its more important features, quantitative, as much so as the general problem of dynamical astronomy; and hence, to that extent, mathematical analysis constitutes the only adequate instrument for its treatment. It is, however, in its entirety, far more complex and much less tractable than the problem of astronomy. The monetary system is not, like the solar system, substantially conservative; it is continually perturbed by psychic forces—the hopes, fears, desires, whims and caprices, the will and conscious purposes of men—forces which are obscure in their origin and not yet expressible in terms of the fundamental units. If, however, at any time we shall succeed in evaluating the effects due to physical causes, the residual effects are plainly referable to these psychic causes, and in a certain sense are their measure.

I purpose to be understood by the many, and hence in the mathematical processes shall sacrifice elegance to perspicuity, and brevity to completeness of exposition. I invite attention to the method employed, to the validity of its principles, to the character of the data, and to the justness of the conclusions. The data are the best that I could obtain at this time, and are to be supplemented and improved by further research. Throughout the paper a year is taken as the unit of time, a ton containing two thousand pounds avoirdupois as the unit of weight, and the sacrifice of a day's manual labor, say with pickaxe and shovel, as the unit of value.

At any designated epoch let

G = the aggregate gold in the world, coined or available for coinage,
S = " silver " " " "
A = the average value of a ton of gold,
B = " " " silver,
$R = A \div B$ = the ratio of silver to gold,
g = the increment of G in one year,
s = " " S " "
r = " " R " "

a = the cost of production of a ton of g,
b = " " " " " s;
and put

$$g \div G = p\,, \quad s \div S = q\,, \quad a \div A = m\,, \quad b \div B = n\,.$$

Then one year after the designated epoch
$G + g$ = the aggregate gold in the world, coined or available for coinage,
$S + s$ = the aggregate silver in the world, coined or available for coinage,
$R + r$ = the new ratio of silver to gold,
$AG + ag$ = the value of $G + g$,
$BS + bs$ = " " $S + s$,
$\frac{AG + ag}{G + g}$ = the average value of a ton of $G + g$,
$\frac{BS + bs}{S + s}$ = " " " " $S + s$,

$$R + r = \frac{AG + ag}{G + g} \div \frac{BS + bs}{S + s}$$

$$= \frac{AG\left[1 + \frac{a}{A}\frac{g}{G}\right]}{G\left[1 + \frac{g}{G}\right]} \div \frac{BS\left[1 + \frac{b}{B}\frac{s}{S}\right]}{S\left[1 + \frac{s}{S}\right]}$$

$$= R \times \frac{(1 + mp)\,(1 + q)}{(1 + p)\,(1 + nq)}$$

= the ratio of silver to gold at the end of the year, *in so far as the variation of that ratio is affected by the supply during the year*. This formula will well repay profound study. The factor

$$\frac{(1 + mp)\,(1 + q)}{(1 + p)\,(1 + nq)}$$

is the important one and may be called the Ratio Factor, and when this is unity the ratio remains constant. For convenience in the numerical calculations I shall throw the formula into another form.

We have by actual division

$$\frac{1}{1 + p} = 1 - p + p^2 - p^3 + \ldots$$

$$\frac{1}{1 + nq} = 1 - nq + n^2q^2 - n^3q^3 + \ldots$$

hence

$$R + r = R(1 + mp)\,(1 + q)\,(1 - p + p^2 - p^3 + \ldots)\,(1 - nq + n^2q^2 - n^3q^3 + \ldots)$$

$$= R + R\,\{q(1 - n) - p(1 - m) - q^2n(1 - n) - qp(1 - n)\,(1 - m) + p^2(1 - m)\}$$

+ other terms too small to require attention.

Subtracting R from each member we get

$$r = R\,\{q(1 - n) - p(1 - m) - q^2n(1 - n) - qp(1 - n)\,(1 - m) + p^2(1 - m)\}$$

for the annual variation of the ratio of silver to gold. When this is zero the ratio remains constant. This formula will repay study. The quantities symbolized by q and p are so small that their squares and their product will practically abolish the terms into which they enter in any applications made in this paper; I shall therefore write the Annual Variation in the simple and instructive form

$$r = R\,\{q(1 - n) - p(1 - m)\}$$

and proceed to the numerical applications. The data are contained in the tables following:

TABLE 1.—*The quantities of gold and silver in the world, coined or available for coinage, and the ratio of wages, estimated in fine gold by weight, at different epochs.*

Year.	Tons.		Ratio of Wages.
	Gold.	Silver.	
1500	.....	.....	7
1600	930	25800	9
1700	1467	50400	20
1800	3058	98600	38
1850	4054	126600	60
1873	7600	150000	82
1890	.....	.....	100

TABLE 2.—*Production of gold and silver in the world, and the commercial ratio of silver to gold, for each calendar year since* 1870.

Year.	Tons.		Ratio.	Year.	Tons.		Ratio.
	Gold.	Silver.			Gold.	Silver.	
1870	.....	.....	15.57	1882	169.1	2965	18.19
1871	.....	.....	15.57	1883	158.2	3058	18.64
1872	.....	.....	15.63	1884	168.7	2798	18.57
1873	159.5	2169	15.92	1885	179.8	3142	19.41
1874	150.5	1896	16.17	1886	175.8	3198	20.78
1875	161.7	2134	16.59	1887	175.5	3295	21.13
1876	172.0	2323	17.88	1888	182.8	3732	21.99
1877	189.1	2148	17.22	1889	204.7	4300	22.09
1878	197.3	2519	17.94	1890	199.7	4608	19.76
1879	180.7	2546	18.40	1891	209.2	4938	20.92
1880	176.6	2563	18.05	1892	.....	.....	23.72
1881	170.8	2705	18.16	1893			

I desire to express every important principle invoked in the evaluations, that nothing may be done covertly, and that everything may contribute, either by its evident truth or by its evident falsity, to the settlement of this intricate question. I hold **that the value of the aggregate stock of gold or of silver is, in the main, and when not demonetized, the equivalent of the aggregate labors and perils undergone in the first acquisition of its several parts.** For example, suppose an ounce of gold dug to-day by the labor of ten men, another ounce dug a thousand years ago to-day by the labor of a hundred men—the ounce dug a thousand years ago contributed ten times as much as the ounce dug to-day to the value of the gold now in the world; and at the same time it is also true that, if both were segregated, the one would be exchanged for the other indifferently. This apparent paradox is due to the reluctance of men to part with gold for less than they gave for it, to the value-sustaining power of monetary use, and to the fact that society is not yet *saturated* with gold. Whenever society shall have attained the condition in which *every one has all the gold that he wants*, the principle will cease to be true. I also hold **that, in the long run, miners in all ages have earned their wages.** Combining this with the foregoing I arrive at the following principle:

> **The contribution, to the present value of the aggregate stock of gold or silver, by an increment to that stock at any epoch, is directly as the weight of the increment, and inversely as the rate of wages, estimated in fine gold or fine silver, obtaining at the same epoch, the wages to be estimated in terms of the metal contributed.**

In using this principle I shall apply both to gold and silver the ratio of wages contained in the last column of Table 1, not going into the refinement of allowing for some small values of the ratio of silver to gold during and prior to the seventeenth century; and I shall, on principle, exclude consideration of the remarkable rise in that value during the past two decades, a preliminary computation having shown that it is due to demonetization and speculation, and hence to be excluded in the evaluation of normal effects. But taking the other and more laborious course would lead to practically the same result.

The annual output of gold given in Table 2 is not all available for coinage. I assume the net annual industrial use to be, gold 100 tons, silver 565 tons, and estimate that 20 tons of this gold and 141 tons of the silver are irrevocably lost to monetary uses. The annual loss by abrasion of coins is approximately, gold 1 ton, silver 80

tons. The net annual flow to the East may be assumed to be, gold 20 tons, silver 1200 tons, of which I will suppose 7 tons of gold and 500 tons of silver to be irrevocably lost to the coinage circulation of the world. This amounts to an irrevocable annual loss to monetary uses of 28 tons of gold and 721 tons of silver, which may be at once deducted from the annual product of those metals. This being done we have Table 3, in which G and S refer to the beginning, and p and q to the middle of the calendar year.

TABLE 3.

Year.	Tons. Gold. g	Silver. s	Gold. G	Silver. S	$p = g \div G$	$q = s \div S$
1873	132	1448	7600	150000	0.0172	0.0096
74	122	1175	7732	151448	157	77
75	134	1413	7854	152623	169	92
76	144	1602	7988	154036	179	103
77	161	1427	8132	155638	196	92
78	169	1798	8293	157065	202	114
79	153	1825	8462	158863	179	115
1880	149	1842	8615	160688	0.0172	0.0114
81	143	1984	8764	162530	162	121
82	141	2244	8907	164514	157	135
83	130	2337	9048	166758	143	139
84	141	2077	9178	169095	152	122
85	152	2421	9319	171172	162	141
86	148	2477	9471	173593	155	142
87	147	2574	9619	176070	151	146
88	155	3011	9766	178644	158	167
89	177	3579	9921	181655	177	195
1890	172	3887	10098	185234	0.0169	0.0208
91	181	4217	10270	189121	174	221
92	.....	.....	10451	193338		

In Tables 4 and 5 I have placed the computation of m and n for the two dates 1873 and 1890, in accordance with the principle enunciated on p. 4.

TABLE 4.

Period.	Increment of Gold.	Ratio of Wages.	Weighted increment.
.....1600	930	÷ 7	= 132.9
1600–1700	537	÷ 14	= 38.4
1700–1800	1591	÷ 28	= 56.8
1800–1850	996	÷ 48	= 20.7
1850–1873	3546	÷ 71	= 50.0
1873–1890	2498	÷ 91	= 27.5

For 1873, $7600 \div 25.4 = 298.8$, $m = \dfrac{25.4}{82} = 0.31$

For 1890, $10098 \div 31.0 = 326.3$, $m = \dfrac{31.0}{100} = 0.31$

TABLE 5.

Period.	Increment of Silver.	Ratio of Wages.	Weighted increment.
.....1600	25800	÷ 7	= 3686
1600–1700	24600	÷ 14	= 1757
1700–1800	48200	÷ 28	= 1721
1800–1850	28000	÷ 48	= 583
1850–1873	23400	÷ 71	= 330
1873–1890	35234	÷ 91	= 387

For 1873, $150000 \div 18.6 = 8077$, $n = \frac{18.6}{82} = 0.23$

For 1890, $185234 \div 21.9 = 8464$, $n = \frac{21.9}{100} = 0.22$

We are now ready to compute the ratio of gold to silver, its annual variation, and the excess of the commercial ratio. I employ for the ratio on Jan. 1, 1883, the value 15.78, which is the mean of the mean values for the years 1872 and 1873 as given on page 162 of the Report of the Director of the Mint for 1892. I do not have the actual ratio. The computed fluctuations throughout the two decades will, however, be practically the same, and all the values may be altered by a small constant quantity, if deemed desirable.

TABLE 6.—*Containing the Annual Variation of the Ratio of Silver to Gold, given by the formula*

$$r = R\{q(1-n) - p(1-m)\}$$

with the Ratio derived therefrom for the years 1874–1892. *In the last column is placed the Excess of the Commercial Ratio for the year.*

Year.	$1-n$	$1-m$	$q(1-n)$	$p(1-m)$	$q(1-n) - p(1-m)$	R Ratio Jan. 1.	r Variation for one year.	Excess of Commercial Ratio.
1873	0.77	0.69	0.0074	0.0119	— 0.0045	15.7	— 0.071	+ 0.18
74	.....	.....	59	108	— 0.0049	15.709	— 0.077	+ 0.50
75	.....	.....	71	117	— 0.0046	15.632	— 0.072	+ 0 99
76	.....	.....	79	124	— 0.0045	15.560	— 0.070	+ 2.36
77	.....	.....	71	135	— 0.0064	15.490	— 0.099	+ 1.78
78	.....	.....	88	139	— 0.0051	15.391	— 0.079	+ 2.59
79	.....	.....	89	124	— 0.0035	15.312	— 0.054	+ 3.12
1880	.....	.....	0.0088	0.0119	— 0.0031	15.258	— 0.047	+ 2.82
81	.....	.....	93	112	— 0.0019	15.211	— 0.029	+ 2.96
82	.....	.....	104	108	— 0.0004	15.182	— 0.006	+ 3.01
83	.....	.....	108	99	+ 0.0009	15.176	+ 0.014	+ 3.46
84	.....	.....	95	105	— 0.0010	15.190	— 0.015	+ 3.39
85	.....	.....	110	112	— 0.0002	15.175	— 0.003	+ 4.24
86	.....	.....	111	107	+ 0.0004	15.172	+ 0.006	+ 5.60
87	.....	.....	114	104	+ 0.0010	15.178	+ 0.015	+ 5.94
88	.....	.....	130	109	+ 0.0021	15.193	+ 0.032	+ 6.78
89	.....	.....	152	122	+ 0.0030	15.225	+ 0.046	+ 6.84
1890	0.78	0.69	0.0162	0.0117	+ 0.0045	15.271	+ 0.069	+ 4.45
91	.....	.....	172	120	+ 0.0052	15.340	+ 0.080	+ 5.54
92	.....	.....	.....	.....	.....	15.420	.....	+ 8.22

Since some of the data used in the foregoing computation is only roughly approximate, it is important to determine to what degree the results are affected by considerable modifications of such data. I have therefore made a second computation in the same way as the first, assuming that the total gold and silver available for monetary uses in 1873 was, gold 7644 tons, silver 154000 tons; that there is lost irrevocably to monetary uses each year—

By industrial use, gold 40 tons, silver 280 tons.

By abrasion of coins, gold 2 tons, silver 55 tons.

By flow to the East, gold 20 tons, silver 900 tons.

The final result is presented in

TABLE 7.—*Results of a second computation with modified data.*

Year.	R Ratio Jan. 1.	r Variation for one year.	Excess of Commercial Ratio.
1873	15.78	— 0.063	+ 0.17
74	15.717	— 0.061	+ 0.48
75	15.656	— 0.067	+ 0.97
76	15.589	— 0.062	+ 2.32
77	15.527	— 0.098	+ 1.74
78	15.429	— 0.079	+ 2.55
79	15.350	— 0.055	+ 3.08
1880	15.295	— 0.047	+ 2.78
81	15.248	— 0.026	+ 2.92
82	15.222	— 0.003	+ 2.97
83	15.219	+ 0.015	+ 3.41
84	15.234	— 0.015	+ 3.34
85	15.219	— 0.005	+ 4.19
86	15.214	+ 0.015	+ 5.56
87	15.229	+ 0.011	+ 5.90
88	15.240	+ 0.032	+ 6.73
89	15.272	+ 0.046	+ 6.79
1890	15.318	+ 0.070	+ 4.41
91	15.388	+ 0.080	+ 5.49
92	15.468	.....	+ 8.21

The necessity of going to press forbids a more elaborate marshaling of the facts at this time. It is my intention, however, to enter upon a much more exhaustive investigation. The first work to be done is the collection of a larger mass of data entitled to confidence; the computations involved in the application of the formula for annual variation are then easily performed. It is hardly necessary to say to mathematicians that I have carefully eschewed the more general methods so delightful to them and so forbidding to the public; the more so that the gist of the whole matter is here presented in much more intelligible form. I now invite attention to important results already attained. An inspection of Tables 6 and 7 (they are

practically the same), in the light of the analysis by which they have been derived from the data, shows that the history of the accumulation of gold and silver, the stock of those metals available for monetary uses in the world, and the annual output, afford no justification or excuse for the demonetization of silver, or for its degradation in the markets of the world. The normal tendency of the ratio of silver to gold was downward, instead of upward, from 1873 to 1886, since which time there has been an upward tendency. But the whole normal fluctuation is quite insignificant when compared with the abnormal and unnatural fluctuation brought about by unwise legislation in this and other countries, by speculation, by the determined and well-directed efforts of the creditor nation, and of the creditor class everywhere, and by the irrational fears of the people. The excess of the commercial ratio exhibited in the last column of Tables 6 and 7 is a measure of the madness of the time. The irregularities of that column (in which, it must be remembered, the numbers stand for the mean of all the values for a year) indicate that it is not the course of nature at all, but of desperate and reckless speculation. The imaginations of men usually distinguished for sobriety of judgment have taken control of them. They see nothing but the "colossal output" of the silver mines, and fear that the whole world is to be drowned in the white metal. Why not subject the matter to calculation? Standing on the bank of the Mississippi one is lost in the contemplation of that mighty volume of water rolling toward the sea; but it may be easily shown that the Mississippi might rise fifty feet and remain there for a year without sensibly affecting the level of the ocean. The colossal annual output of both gold and silver mines is going to join the still more colossal aggregate output of centuries. This vast aggregate is the dominant fact, acting as a conservative reservoir to prevent the mischiefs of monetary fluctuations. When we come to know this, to realize this, we will utilize the fact and make our laws accord with and reflect the wisdom of nature. Subjected to rigorous computation, where each element is given its just weight, the result is, that the normal ratio of silver to gold is at present about $15\frac{1}{2}$, and the American silver dollar is worth about $1.03 in gold.

WASHINGTON, *Aug.* 24, 1893.

Note added Aug. 25th.—I have prepared a much more lucid and intelligible demonstration without the employment of algebra, which will be published within a few days. The result is substantially the same.

END OF

TITLE